STUDY OF SELECTED FISSION REACTIONS WITH THE APPLICATION OF NILSSON ORBITALS

by

Aziz N. BEHKAMI* and Soleiman RASOULI

Department of Physics, Mahabad Branch, Islamic Azad University, Mahabad, Iran

Short paper
UDC: 539.125.5:519.218
DOI: 10.2298/NTRP1103245B

Fission fragment angular anisotropies from neutron induced fission of ^{232}Th and ^{235}U were analyzed within the frame work of the statistical model. The analysis were made at neutron energies from threshold up to 50 MeV to deduce the variance K_0^2 of the *K*-distribution of levels in the transition nucleus. Our analysis shows, that the strength for the *K*-transition states comes mainly from the higher angular momentas and is in accordance with Nilsson model orbitals.

Key words: fission angular anisotropies, ^{232}Th(n, f) and ^{238}U(n, f) fission reactions, various neutron energies, Nilsson orbitals at different excitations

INTRODUCTION

At neutron energies, well above the threshold for even-even target nuclei, the channels for fission open fully, and many *K*-values become possible. This happens at excitation energies above 1.15 MeV, when it becomes possible to separate neutron pairs, and the complexity of the transition state spectrum increases rapidly. So, at higher excitation energies fission is expected to occur through channels defined by *K*-values distributed according to a Gaussian distribution centered around $K = 0$. It is thus appropriate to take the statistical distribution of channels at these energies. The variance of the distribution K_0^2 is expected to vary with energy, since as the energy increases, more channels will become accessible and K_0^2 will change accordingly. This will make it possible to get information about the contribution of Nilsson levels at different excitations. The fission angular distribution data are evidently crucial to obtain information on the variance K_0^2. The details of the theory are described in the Section on theoretical formalism. Results and discussion will be presented in the last section.

THEORETICAL FORMALISM

The probability that a compound nucleus will decay through a transition state is proportional to the density of levels (I, K) in the transition state nucleus. This is given by [1]

$$\rho(J,K) \quad \exp\frac{E \quad E_{\mathrm{rot}}^{I,\pi,K}}{T} \tag{1}$$

where E is the total energy of the nucleus, $E_{\mathrm{rot}}^{I,\pi,K}$ is the rotational energy of the level (I, , K) in a transition state, and T is the nuclear temperature which is a measure of the extent to which nucleons occupy energy levels above the Fermi energy. The rotational energy is given by [2]

$$E_{\mathrm{rot}}^{I,\pi,K} \quad \frac{\hbar^2}{2}(I^2 \quad K^2) \quad \frac{\hbar^2}{2_{\parallel}}K^2 \tag{2}$$

From eqs. (1) and (2) one obtains

$$\rho(I,\pi,K) \quad \exp\ \frac{E}{T} \quad \frac{\hbar^2 I^2}{2 \quad T} \quad \frac{\hbar^2 K^2}{2T} \quad \frac{1}{\ _{\parallel}} \quad \frac{1}{\ } \tag{3}$$

For fixed values of E, T, and I it becomes

$$\rho(I,\pi,K) \quad \exp \quad \frac{\hbar^2 K^2}{2T} \quad \frac{1}{\ _{\parallel}} \quad \frac{1}{\ } \tag{4}$$

This equation is equivalent to a Gaussian *K*-distribution and can be written as

$$\rho(I,\pi,K) \quad \exp\frac{K^2}{2K_0^2} \tag{5}$$

where $K_0^2 = (T/\hbar^2)[(1/\ _{\parallel}) - (1/\Im\ )]$ represents the variance of the Gaussian *K*-distribution of transition states.

The excited levels in the transition nucleus are characterized by *K* quantum number which is the projection of total angular momentum along the symmetry axis. With the assumption that the fragments separate along the symmetry axis and that *K* is a good quantum number during fission, the fragment angular

* Corresponding author; e-mail: behkami@susc.ac.ir

distribution from a state with quantum numbers K and M (projection of total angular momentum along the space fixed axis) is given by [3]

$$W^I_{M,K}(\theta) = \frac{2I+1}{4\pi}\left|d^I_{M,K}(\theta)\right|^2 \tag{6}$$

The normalized $d^I_{M,K}(\theta)$ functions are defined by [4]

$$d^I_{M,K}(\theta) = \sqrt{(I+M)!(I-M)!(I+K)!(I-K)!}\sum_X \frac{(-1)^X \left(\sin\frac{\theta}{2}\right)^{K-M+2X}\left(\cos\frac{\theta}{2}\right)^{2I-K+M-2X}}{(I-K-X)!(I+M-X)!(X+K-M)!X!} \tag{7}$$

where the sum is over $X = 0, 1, 2, ...$ and contains all terms in which no negative value appears in the denominator of the sum for any quantity in parentheses.

If the target and the projectile spins are zero and there is no particle emission from the initial compound nucleus before fission ($i.e.\ M = 0$), then the overall angular distribution for a fixed energy E, is given by [5, 6]

$$W(\theta) = \frac{\sum_{J=0}^{\infty}(2I+1)T_I \sum_{K=-I}^{I}(2I+1)\left|d^I_{M=0,K}(\theta)\right|^2 \exp\left(-\frac{K^2}{2K_0^2}\right)}{\sum_{K=-I}^{I}\exp\left(-\frac{K^2}{2K_0^2}\right)} \tag{8}$$

where the transmission coefficients are written as T_l, since $\ell = I$ when $M = 0$. Equation (8) is an exact theoretical expression for computation of fission fragment angular distribution when both the target and projectile spins are zero. If the target and projectile spins are included, an exact expression for the fission fragment angular distribution is [7]

$$W(\theta) \propto \sum_{I=0}^{\infty}\sum_{M=-j_{max}}^{j_{max}}\sum_{\ell=0}^{\infty}\sum_{j=|I_0-s|}^{I_0+s}\sum_{\mu=-I_0}^{I_0} f_{I,M,\ell,j,\mu}\sum_{K=-I}^{I} g_{I,M,K} \tag{9}$$

where

$$f_{I,M,\ell,j,\mu} = \frac{(2\ell+1)T_\ell\left|C^{j,\ell,I}_{M,0,M}\right|^2\left|C^{I_0,s,j}_{\mu,M-\mu,M}\right|^2}{\sum_{\ell=0}^{\infty}(2\ell+1)T_\ell}$$

and

$$g_{I,M,K} = \frac{(2I+1)\left|d^I_{M,K}(\theta)\right|\exp\left(-\frac{K^2}{2K_0^2}\right)}{\sum_{K=1}^{I}\exp\left(-\frac{K^2}{2K_0^2}\right)}$$

The quantity I_0, s, and j, are the target spin, projectile spin and channel spin, respectively. The channel spin j is defined by the relation $j = I_0 + s$. The total angular momentum I is given by the sum of the channel spin and orbital angular momentum; $I = j + \ell$. The projection of I_0 on the space-fixed axis is given by μ, whereas the projection of j (and I) on this axis is M.

The use of eqs. (8) or (9), requires the evaluation of many $d^I_{M,K}(\theta)$ functions and the Clebsch-Gordan coefficients, hence these equations have rarely been used for data analysis. In the present paper, we have developed a special computer code to run these more cumbersome theoretical expressions and thereby to deduce the statistical variance K_0^2. In the following we represent the obtained quantitative values of the variance K_0^2.

RESULTS AND DISCUSSION

Fragment angular distribution data at limited energy ranges have been compiled by various groups [8, 9]. Fragment anisotropy data for ^{232}Th(n, f) and ^{238}U(n, f) fission reactions [10, 11] have been analyzed and the statistical variance K_0^2 has been obtained by fitting the experimental fragment anisotropies with exact theoretical expressions (8) and (9). Optical model transmission coefficients have been used in the calculations.

The curve in fig. 1 illustrates the theoretical dependence of K_0^2 on the fission anisotropy. From this figure it can be seen that K_0^2 - parameter becomes larger at smaller anisotropies. Calculations of the variance K_0^2 for ^{232}Th(n, f) system at various neutron energies,

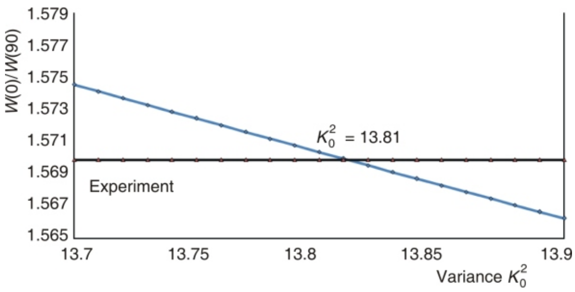

Figure 1. Fragment anisotropy $W(0)/W(90)$ of fission fragments for ^{232}Th(n, f) system at 21 MeV neutrons. The theoretical curve is calculated using eq. (9)

Table 1. Values of the anisotropies for ^{232}Th(n, f) reaction together with the deduced values of the variances K_0^2

Neutron energy [MeV]	Anisotropy $W(0)/W(\theta)$	Variance K_0^2
2.3	1.75	1.47
2.5	1.66	2.47
3.0	1.40	3.77
4.0	1.16	11.96
6.0	1.12	21.97
7.0	2.42	2.15
8.0	2.41	2.48
10.0	1.79	5.91
12.0	1.47	10.94
13.0	1.52	10.56
14.0	1.68	8.90
15.0	1.70	8.34
16.0	1.52	12.79
18.0	1.79	9.03
20.0	2.10	6.82

have been made using the exact eq. (9). The optical model transmission coefficients have been used again in all calculations. The results are listed in tab. 1 together with the experimental fragment anisotropies for this reaction.

The best fit values of K_0^2 listed in tab. 1, are also plotted in fig. 2 as a function of incident energy. Figure 2 shows that the variance K_0^2 increases smoothly with neutron energy. This behavior implies that as the excitation energy increases, many more single particle Nilsson levels contribute and cause the population of K-states to become large. Similar analysis has been made for the ^{238}U(n, f) system. For example, the dependence of K_0^2 on energy is shown in fig. 3. We see that the K_0^2 parameter increases again for smaller anisotropies. The best fit values of the variance of the K-distribution for various neutron energies have been computed for ^{238}U(n, f) system in the same way, as it was done for ^{232}Th(n, f) reactions. The exact theoretical eq. (9) was used in the calculations. The results are listed in tab. 2 and are plotted in fig. 4. In particular, smooth increase of K_0^2 with incident energy indicates the contributions of many Nilsson levels in this region.

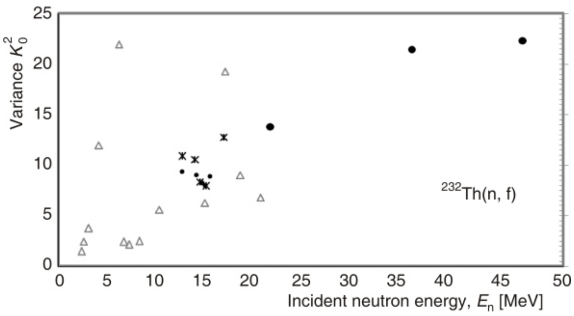

Figure 2. Dependence of the variance K_0^2 on neutron energy for ^{230}Th(n, f) fission reaction. Note the smooth increase in K_0^2 with increasing incident neutron energy. Symbols represent the experimental results of different authors [11-13]

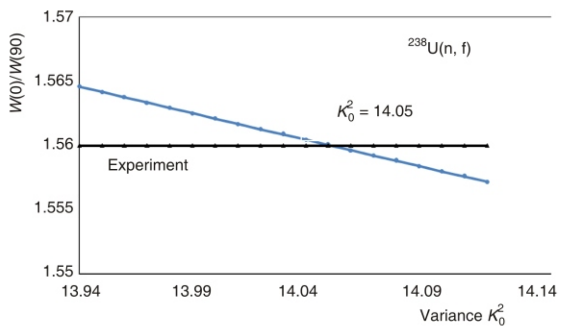

Figure 3. Fragment anisotropy $W(0)/W(90)$ of fission fragments for ^{238}U(n, f) system at 21 MeV neutrons. The theoretical curve is calculated using eq. (8)

Table 2. Values of the anisotropies for ^{232}U(n, f) reaction together with the deduced values of the variances K_0^2

Neutron energy [MeV]	Anisotropy $W(0)/W(\theta)$	Variance K_0^2
1.0	1.53	1.15
1.4	1.55	1.68
2.3	1.34	3.66
3.0	1.34	4.41
4.0	1.28	6.86
5.0	1.26	8.96
5.8	1.38	7.10
13.5	1.38	14.30
14.0	1.55	10.42
15.0	1.46	13.02
21.0	1.56	14.05
35.0	1.20	37.20
46.0	1.41	33.80

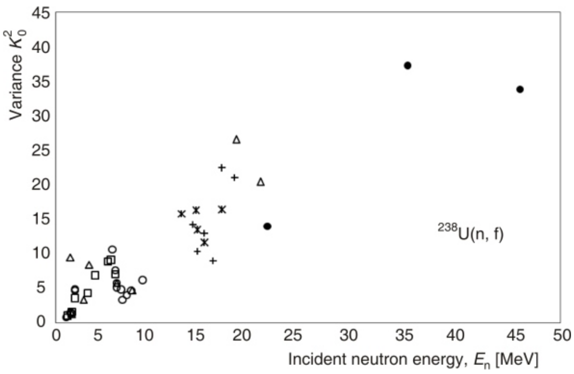

Figure 4. Dependence of the variance K_0^2 on neutron energy for ^{238}Th(n, f) fission reaction. Note the smooth increase in K_0^2 with increasing incident neutron energy. Symbols represent the experimental results of different authors [11-13]

ACKNOWLEDGEMENTS

We would like to express our appreciation to the Islamic Azad University, Fars Science and research branch, Shiraz, for their assistance during the course of this work. This work was performed under the auspicious of the Islamic Azad University, Mahabad Branch.

REFERENCES

[1] Vandenbosch, R., Huizenga, J. R., Nuclear Fission, Academic press, New York, USA, 1973, pp. 414-416
[2] Datta, T., *et al.*, Angular Distribution in Alpha Induced Fission of ^{232}Th and ^{238}U, *Physical Rev. C, 48* (1993), pp. 221-227
[3] Behkami, A. N., *et al.*, Statistical Treatment of Fission Fragment Produced in Heavy-Ion Reactions, *Nucl. Sci. Journal, 37* (2000), 6, pp. 403- 408
[4] Behkami, A. N., Nuclear Data A Tables, Vol. 10, No. 1, 1971, pp. 1-47
[5] Rahimi, F. M., Ghodsi, O. N., Behkami, A. N., Influence of Spin on Fussion Fragment Anisotropy, *Nuclear Technology & Radiation Protection, 20* (2005), 1, pp. 45-49

[6] Behkami, A. N., Nazarzadeh, P., Study of Fragment Angular Anisotropies and Inertia Parameters, *Nucl. Sci. Journal, 9* (1998), 2, pp. 193-206
[7] Blons, J., Mazor, C., Paya, D., Fission Cross-Section and Angular Distribution in ^{232}Th(n, f) Reaction, *Phys. Rev. Lett., 35* (1975), p. 1749
[8] Becker, J. A., Bauer, R. W., Fragment Angular Distributions of Neutron Fission of ^{232}Th, *Phys. Rev. C., 34* (1986), pp. 394-400
[9] El-Hajje, R., A Simultaneous Measurement of the Angular Distribution Mass and Kinetic Energy of ^{235}U and ^{232}Th Fission Fragments, Ph. D. thesis, School of Safety Sciences, University of New South Wales, Australia, 2000, Sec. III
[10] Rahimi, F. M., Ghodsi, O. N., Behkami, A. N., Angular Distribution and Inertia Parameters in Alpha Induced Fission of ^{232}Th , ^{235}U, and ^{238}U, *Nuclear Technology & Radiation Protection, 18* (2003), 1, pp. 31-35
[11] Ryzhov, I. V., *et al*., Influence of Multichance Fission on Fragment Angular Anisotropy in the ^{232}Th(n, f), and ^{238}U(n, f) Reaction at Intermediate Energies, *Nuclear Physics A, 760* (2005), 1-2, pp. 19-39
[12] Shpak, D. L., *et al*., Angular Anisotropy of the Fragments from Fission by 0.85-6.28 MeV Neutrons, *Yad. Fiz., 50* (1989), 4, pp. 922-927
[13] Henkel, R. L., Brolley, Jr. J. E., Angular Distribution of Fragments from Neutron-Induced Fission of ^{238}U and ^{232}Th, *Phys. Rev., 103* (1956), pp. 1292-1295

Received on: August 24, 2011
Accepted on: October 21, 2011

Азиз Н. БЕХКАМИ, Солејман РАСОУЛИ

ПРОУЧАВАЊЕ ИЗАБРАНИХ ФИСИОНИХ РЕАКЦИЈА ПРИМЕНОМ НИЛСОНОВИХ ОРБИТАЛА

Угаоне анизотропије фисионих фрагмента насталих фисијама ^{232}Th и ^{235}U изазваним неутронима разматране су у оквирима статистичког модела. Анализе су спроведене за енергије неутрона од прага за фисију до 50 MeV, да би се дедуковала K_0^2 варијанса K-расподела нивоа у прелазним језгрима. Наша анализа показује да енергија за K-транзициона стања потиче углавном од виших угаоних момената и да је у складу са Нилсоновим моделом орбитала.

Кључне речи: фисиона угаона анизотропија, $^{232}Th(n,f)$ и $^{238}U(n,f)$ реакције фисије, неутронска енергија, Нилсонове орбитале